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Transparency windows for plasma waves on the surface of the nanotube with a superlattice

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Within the framework of the model electron energy spectrum on the nanotube surface with a superlattice in a magnetic field, an exact expression for the polarization operator of a degenerate electron gas was obtained. The shape and size of the plasma waves Landau damping regions on the tube throughout the Brillouin zone were calculated. The influence on these areas of the position of Fermi level in the miniband was considered. The conditions for the resonance absorption of plasmon on the tube by electrons were found. The limiting transition towards the nanotube without superlattice was performed.
Keywords: nanotube, superlattice, magnetic field, plasma waves, Landau damping.

На основе модельного спектра энергии электронов на поверхности нанотрубки со сверхрешеткой в магнитном поле получено точное выражение для поляризационного оператора вырожденного электронного газа. Рассчитаны форма и размеры областей затухания Ландау плазменных волн на трубке во всей зоне Бриллюэна. Рассмотрено влияние на эти области положения уровня Ферми в минизоне. Найдены условия резонансного поглощения плазмонов на трубке электронами. Выполнен предельный переход к нанотрубке без сверхрешетки.
Ключевые слова: нанотрубка, сверхрешетка, магнитное поле, плазменные волны, затухание Ландау.

На основі модельного спектру енергії електронів на поверхні нанотрубки з надграткою у магнітному полі отримано точний вираз для поляризаційного оператора виродженого електронного газу. Розраховані форма і розміри областей згасання Ландау плазмових хвиль на трубці у всій зоні Бриллюена. Розглянуто вплив на ці області положення рівня Фермі у мінізоні. Знайдені умови резонансного поглинання плазмонів на трубці електронами. Виконаний граничний перехід до нанотрубки без надгратки.
Ключові слова: нанотрубка, надгратка, магнітне поле, плазмові хвилі, згасання Ландау.

Introduction

After the emergence of the idea of Keldysh [1] on the establishment of an additional crystal periodic potential and its realization in layered semiconductor structures [2,3] have passed fifty years. However, interest in this system, called the superlattices (SL), continues unabated. The number of articles, reviews and monographs on semiconductor superlattice is steadily increasing [4-15]. The suggested by Keldysh perspective to control band spectrum of a solid proved tempting. The technical applications of SL is very broad . In recent years the physics of solid state enriched set of effects and technical applications due to study of SL.

In Refs. [5,6] the de Haas-van Alphen effect in layered conductors with additional periodic potential which perpendicular to the layers changes.

In Ref. [11] the electromagnetic waves in the SL in a magnetic field were considered. The spectrum of low-frequency oscillations of the field in conditions of the quantum Hall effect was obtained. This effect is observed experimentally in SL GaAs / (AlGa) in magnetic field which perpendicular to the axis of the superlattice. The undamped helicons in SL were predicted. They do not damped because the dissipative components of the conductivity tensor under certain values of the magnetic field strength equals zero. Besides, under the frequencies of fields which less than the electron cyclotron frequency, there is no Landau damping. Thanks to the non-dissipative motion of electrons in the SL the spectrum and polarization of electromagnetic waves in it are of an unusual character.

In Ref. [12] it is shown that in the SL in a magnetic field it is possible metal-insulator transition due to the dependence of the density of states of electrons on the magnetic field. This article calculates the magnetic field values at which the dissipative components of the conductivity tensor of the electron gas vanish. The dependence of the photoconductivity of SL on the magnetic field was considered. In Ref. [13] the thermodynamic functions of the SL in the magnetic field for the degenerate and nondegenerate electron gas were calculated. A new method of adiabatic cooling of the sample was suggested.

In Refs. [2-15] the one-dimensional superlattice with additional periodic potential depending on one coordinate were considered. The new surge of interest in CP associated with the discovery of carbon [16] and semiconductor [17] nanotubes. There was a need to consider the SL on the surface of the tube. The SL with cylindrical symmetry may be radial and longitudinal. The radial SL is a set of coaxial cylinders. The longitudinal SL is similar to the system of coaxial rings strung on the axis of the cylinder. Such SL can exist on a tube filled with fullerenes or metal atoms [18-20].

Landau damping of plasma waves in the degenerate and nondegenerate electron gas on the surface of the nanotubes in a longitudinal magnetic field in the absence of the superlattice is considered in the paper [21]. The authors of this paper calculated the longitudinal dielectric function of the electron gas on the tube. Restricting ourselves investigation taken into account only the intraband transitions of electrons, they found the Aharonov-Bohm oscillations of the dielectric constant.

In Refs. [22,23] the long-wave plasma oscillations on the tube with a superlattice were considered. In these articles, the transparency windows for plasma waves is considered only for certain values of the wave vector and the frequency of the waves. Here, within the established in the article [22] model, we present the results of calculating the position and shape of the region of Landau damping plasma waves in the whole plane wavenumber-frequency.

The polarization operator of the electron gas

The energy of the electron with the effective mass m_* on the surface of a cylindrical nanotube in a magnetic field parallel to its axis, is calculated by Kulik taking into account the quantization of radial motion of electrons in a tube of small thickness [24]:

$$\varepsilon_{lk} = \varepsilon_0 (l+\eta)^2 + \frac{\hbar^2 k^2}{2m_*}, \tag{1}$$

where $\hbar l$ and $\hbar k$ – the projection of the angular momentum and momentum of electron on the tube axis, $\varepsilon_0 = \hbar^2 / 2m_* a^2$ – rotational quantum, a – the radius of the tube, $\eta = \Phi / \Phi_0$ – the ratio of the magnetic flux Φ through the cross section of the tube to the flux quantum $\Phi_0 = 2\pi c\hbar / e$ [24]. The equation (1) describes a set of one-dimensional adjoining subbands whose boundaries $\varepsilon_l = \varepsilon_0 (l+\eta)^2$ coincide with the quantized energy levels of the circular motion of the electrons on the tube in the magnetic field. The electron density of states has a root singularity at the boundary of the subzone. The simplest way to take into account the superlattice on the tube is to replace the energy of the longitudinal motion of the electron in the formula (1) to the expression

$$\varepsilon_k = \Delta(1-\cos kd). \tag{2}$$

This expression is borrowed from the theory of the strong coupling of electrons with the lattice, and is often used in the theory of superlattices and layered crystals [15,25,26]. As a result of such a substitution the electron spectrum on the tube becomes

$$\varepsilon_{lk} = \varepsilon_0 (l+\eta)^2 + \Delta(1-\cos kd), \tag{3}$$

where d – period of superlattices, 2Δ – band width in the energy spectrum of the longitudinal motion of the electron. This band corresponds to the values of the wave number k in the first Brillouin zone $-\pi/d \le k \le \pi/d$. The spectrum (3) describes the set of allowed energy region of electron inside the intervals $\varepsilon_l \le \varepsilon \le 2\Delta$, separated by gaps. By analogy with traditional superlattice these bands are called minibands. The electron density of states has a root singularity at the miniband boundaries [27].

In the random phase approximation the damping decrement of plasma waves with angular momentum $\hbar m$ and the wave number q on the tube is equal to [22]

$$\gamma_m(q) = \frac{\operatorname{Im} P_m(q,\omega)}{\frac{\partial}{\partial\omega} \operatorname{Re} P_m(q,\omega)} \Bigg|_{\omega=\omega_m(q)}, \tag{4}$$

where $P_m(q,\omega)$ – the polarization operator of electron gas, $\omega_m(q)$ – plasmon spectrum. The polarization operator is equals

$$P_m(q,\omega) = \frac{1}{\pi a L} \sum_{lk} \frac{f\left(\varepsilon_{(l+m)(k+q)}\right) - f(\varepsilon_{lk})}{\varepsilon_{(l+m)(k+q)} - \varepsilon_{lk} - \hbar\omega - i0}, \tag{5}$$

where $f(\varepsilon)$ – Fermi function, L – the length of the tube.

The dispersion equation for the spectrum and damping of plasma waves has the form

$$1-\upsilon_m(q)P_m(q,\omega)=0, \tag{6}$$

where

$$\upsilon_m(q)=4\pi e^2 a \mathrm{I}_m(qa)\mathrm{K}_m(qa) \tag{7}$$

– cylindrical harmonic of electron Coulomb potential on the tube, I_m and K_m – modified Bessel functions, e – the electron charge.

If the electron gas is degenerate, the integration with respect k in the formula (5) is limited to an interval $[-k_l, k_l]$, where

$$k_l=\frac{1}{d}\arccos\left(\frac{\varepsilon_l+\Delta-\mu_0}{\Delta}\right) \tag{8}$$

– the maximum wave number of electrons in a partially filled miniband with the number l, μ_0 – the Fermi energy. The resulting integration over the k the l-miniband contribution to the real part of the polarization operator is defined by parameters

$$c_\pm=\frac{\hbar(\omega-\Omega_\pm)}{2\Delta\sin\frac{qd}{2}}. \tag{9}$$

Here

$$\hbar\Omega_\pm(l,m)=\varepsilon_0 m\left[2(l+\eta)\pm m\right], \tag{10}$$

Ω_+ – the frequency of direct transitions of electrons $l\to l+m$ between the miniband of the spectrum (3).

If $c_\pm^2<1$, from the formula (5) we obtain

$$\begin{aligned}
\operatorname{Re}P_{lm}(q,\omega)=&-\frac{1}{4\pi^2 ad\Delta\sin\frac{qd}{2}}\\
&\left\{\frac{1}{\sqrt{1-c_+^2}}\left[\ln\left|\frac{c_+\mathrm{tg}\frac{1}{2}\left(x_l+\frac{qd}{2}\right)-\left(1-\sqrt{1-c_+^2}\right)}{c_+\mathrm{tg}\frac{1}{2}\left(x_l+\frac{qd}{2}\right)-\left(1+\sqrt{1-c_+^2}\right)}\right|-\right.\right.\\
&\left.-\ln\left|\frac{c_+\mathrm{tg}\frac{1}{2}\left(-x_l+\frac{qd}{2}\right)-\left(1-\sqrt{1-c_+^2}\right)}{c_+\mathrm{tg}\frac{1}{2}\left(-x_l+\frac{qd}{2}\right)-\left(1+\sqrt{1-c_+^2}\right)}\right|\right]-\\
&-\frac{1}{\sqrt{1-c_-^2}}\left[\ln\left|\frac{c_-\mathrm{tg}\frac{1}{2}\left(x_l-\frac{qd}{2}\right)-\left(1-\sqrt{1-c_-^2}\right)}{c_-\mathrm{tg}\frac{1}{2}\left(x_l-\frac{qd}{2}\right)-\left(1+\sqrt{1-c_-^2}\right)}\right|-\right.\\
&\left.\left.-\ln\left|\frac{c_-\mathrm{tg}\frac{1}{2}\left(-x_l-\frac{qd}{2}\right)-\left(1-\sqrt{1-c_-^2}\right)}{c_-\mathrm{tg}\frac{1}{2}\left(-x_l-\frac{qd}{2}\right)-\left(1+\sqrt{1-c_-^2}\right)}\right|\right]\right\},
\end{aligned} \tag{11}$$

where $x_l=k_l d$. When the electrons are completely filled the l-miniband, in the formula (11) $x_l=\pi$.

In case of $c_\pm^2>1$ we find

$$\begin{aligned}
\operatorname{Re}P_{lm}(q,\omega)=&+\frac{1}{2\pi^2 ad\Delta\sin\frac{qd}{2}}\\
&\left\{\frac{1}{\sqrt{c_+^2-1}}\left[\operatorname{arctg}\frac{c_+\mathrm{tg}\left(x_l+\frac{qd}{2}\right)-1}{\sqrt{c_+^2-1}}+\right.\right.\\
&\left.\operatorname{arctg}\frac{c_+\mathrm{tg}\left(x_l-\frac{qd}{2}\right)+1}{\sqrt{c_+^2-1}}\right]-\\
&-\frac{1}{\sqrt{c_-^2-1}}\left[\operatorname{arctg}\frac{c_-\mathrm{tg}\left(x_l+\frac{qd}{2}\right)-1}{\sqrt{c_-^2-1}}+\right.\\
&\left.\left.+\operatorname{arctg}\frac{c_-\mathrm{tg}\left(x_l-\frac{qd}{2}\right)+1}{\sqrt{c_-^2-1}}\right]\right\}.
\end{aligned} \tag{12}$$

If we restrict our consideration only the intraband transitions ($m=0$, $\Omega_\pm=0$, $c_+=c_-=c=\hbar\omega\Big/2\Delta\sin\frac{qd}{2}$), the formulas (11) and (12) is simplified.

Transition in the formulas (11) and (12) towards to the nanotube without superlattice is performed according to the rule

$$d\to 0,\ \ \Delta\to\infty,\ d^2\Delta\to \hbar^2\big/m_* . \tag{13}$$

In this case, the spectrum (3) becomes (1) and formula (11) takes the form

$$\begin{aligned}
\operatorname{Re}P_{lm}(q,\omega)=&-\frac{m_*}{2\pi^2\hbar^2 qa}\times\\
&\times\left[\ln\left|\frac{\omega-(\Omega_++q\upsilon_l+\omega_q)}{\omega-(\Omega_+-q\upsilon_l+\omega_q)}\right|-\right.\\
&\left.-\ln\left|\frac{\omega-(\Omega_-+q\upsilon_l-\omega_q)}{\omega-(\Omega_--q\upsilon_l-\omega_q)}\right|\right],
\end{aligned} \tag{14}$$

where

$$\upsilon_l = \sqrt{\frac{2}{m_*}}\sqrt{\mu_0 - \varepsilon_l}$$

-maximal velocity of the electrons in the l-th subzone without superlattice, $\omega_q = \hbar q^2 / 2m_*$.

In the absence of interband transitions ($m = 0$) from the formula (5) we obtain the contribution of the l-miniband into the imaginary part of the polarization operator:

$$\operatorname{Im} P_{l0}\left(q,\omega\right) = -\frac{1}{4\pi a d\Delta\left|\sin\frac{qd}{2}\right|\sqrt{1-c^2}}, \tag{15}$$

where $q < 2k_l$, $0 < \omega < 2\frac{\Delta}{\hbar}\sin\frac{qd}{2}\sin\left(x_l + \frac{qd}{2}\right)$.

If q k, the function $\operatorname{Im} P_{l0}$ is still equal to (15), however $\omega_- < \omega < \omega_+$, where

$$\omega_\pm = 2\frac{\Delta}{\hbar}\sin\frac{qd}{2}\sin\left(\pm x_l + \frac{qd}{2}\right). \tag{16}$$

Fig. 1 shows graphs of functions (15) in these cases.

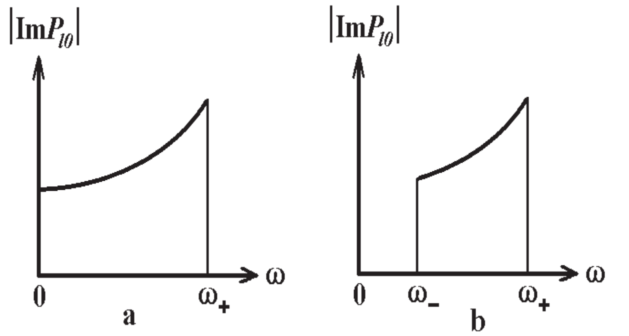

Fig. 1. The dependence of the imaginary part of polarization operator (15) as the functions of the frequency at $q < 2k_l$ (a), $q > 2k_l$ (b).

The values of the jump in the points $\omega_\pm$ are

$$\frac{1}{4\pi a d\Delta\left|\sin\frac{qd}{2}\cos\left(\pm x_l + \frac{qd}{2}\right)\right|}.$$

Taking into account the interband transitions ($m \neq 0$) instead of formula (15) in the vicinity of the frequencies $\Omega_\pm$ we get

$$\operatorname{Im} P_{lm}^{\pm}\left(q,\omega\right) = -\frac{1}{4\pi a d\Delta\left|\sin\frac{qd}{2}\right|\sqrt{1-c_\pm^2}}, \tag{17}$$

as in the formulas (16) terms $\Omega_\pm$ appear.

In the absence of spatial dispersion from the formula (5) at any temperature we obtain

$$\operatorname{Im} P_m\left(\omega\right) = \frac{\pi}{\hbar}\sum_l n_l\left[\delta\left(\omega - \Omega_-\right) - \delta\left(\omega - \Omega_+\right)\right], \tag{18}$$

where n_l – the surface density of electrons in the l-th miniband. In this case, the imaginary part of the polarization operator has sharp jump at frequencies of direct electron transitions between minibands.

Transparency windows

From formula (4) it is seen that for a obtaining of transparency windows for plasma waves in a degenerate electron gas on the surface of the nanotubes it is sufficient to consider the regions on a plane $q-\omega$, in which $\operatorname{Im} P = 0$. The same regions can be found from the laws of conservation of angular momentum projection and electron momentum projection on the axis of the tube, and from the law of conservation of energy under the electron-plasmon absorption. These conservation laws are:

$$\varepsilon_{lk} + \hbar\omega - \varepsilon_{(l+m)(k+q)} = 0. \tag{19}$$

From formula (5) we have seen that the left-hand side of equation (19) is the argument of the delta function, including into the imaginary part of the polarization operator. In addition, when an quantum electron transfer $\left(l,k\right) \to \left(l+m, k+q\right)$ is occurs, involving the absorption of a plasmon, the Pauli principle must be performed: $\varepsilon_{lk} < \mu_0 < \varepsilon_{(l+m)(k+q)}$. Consequently, after the substitution $k = \pm k_l$ in the equation (19), we obtain the boundaries of collisionless damping of plasma waves on the tube with a superlattice in a magnetic field:

$$\omega_\pm\left(q\right) = \Omega_\pm + 2\frac{\Delta}{\hbar}\sin\frac{qd}{2}\sin\left(\pm x_l + \frac{qd}{2}\right). \tag{20}$$

These equations contains the value $x_l = k_l d$ which determines the position of the Fermi level μ_0 in the l-th miniband. From the formula (8) it follows that under $x_l = 0$ Fermi energy is located at the "bottom" of the miniband ($\mu_0 = \varepsilon_l$). With the growth of the x_l Fermi energy and the electron density increases. When $x_l = \pi/2$ the level μ_0 is located in the center of the miniband ($\mu_0 = \varepsilon_l + \Delta$) and reaches her "ceiling" on the boundary of the Brillouin zone ($x_l = \pi,\ \mu_0 = \varepsilon_l + 2\Delta$). Thus, when $0 < x_l < \pi/2$ the Fermi level is located in the lower half of the miniband, while $\pi/2 < x_l < \pi$ – at the top.

When $q \to 0$, the difference $\omega_+ - \omega_-$ in the vicinity of each frequency $\Omega_\pm$ decreases, the Landau damping region is narrowed in accordance with the behavior of the polarization operator (18) in the absence of spatial dispersion. This narrowing is occurs at the boundary of the Brillouin zone where the second term on the right-hand side of (20) is equal to

$$\omega_\pm - \Omega_\pm = 2\frac{\Delta}{\hbar}\cos x_l . \qquad (21)$$

The shape and dimensions of the regions of Landau damping in the vicinity of the frequencies $\Omega_\pm$ are determined by the position of the Fermi level in the miniband. When μ_0 increases from the "bottom" of the l-th miniband to its "ceiling" the expression (21) in the vicinity of the frequencies $\Omega_\pm$ decreases from $2\Delta/\hbar$ to $-2\Delta/\hbar$.

Figure 2 schematically shows Landau damping region between the curves (20) for various locations of the Fermi energy in the miniband in the vicinity of the frequency Ω_+. Outside these regions until the curves

$$\omega_{\min} = \Omega_+ - 2\frac{\Delta}{\hbar}\sin\frac{qd}{2},\quad \omega_{\max} = \Omega_+ + 2\frac{\Delta}{\hbar}\sin\frac{qd}{2} \qquad (22)$$

located transparency window for plasma waves. The curves of (22) – solution of the equation $|c_+| = 1$. When

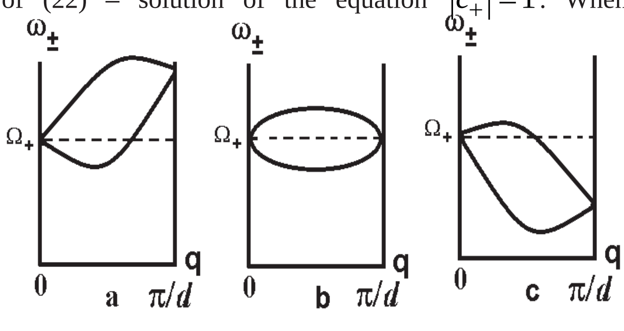

Fig. 2. Areas of Landau damping between the curves (20) at $\cos x_l > 0$ (a), $\cos x_l = 0$ (b) and $\cos x_l < 0$ (c).

$\hbar\Omega_+ < 2\Delta$, the graph of the curve $\omega_{\min}(q)$ (22) intersects the axis q at the point

$$q_0 = \frac{2}{d}\arcsin\frac{\hbar\Omega_+}{2\Delta}.$$

This point tends to the boundary of the Brillouin zone, when $\hbar\Omega_+ \to 2\Delta$. If $\hbar\Omega_+$ greater than the width of the miniband 2Δ, there is no intersection, i.e. $\omega_-(q) > 0$ located in the Brillouin zone.

The areas of Landau damping in the vicinity of the frequency Ω_- are similar to those shown in Fig. 2. Note, that when $\eta > 1/2$ the boundaries of the miniband are satisfy of inequalities $\varepsilon_{-1} < \varepsilon_0 < \varepsilon_{-2} < \ldots$ In this case, the frequency of the direct transition of electrons $-1 \to -2$ with $m = -1$ is equal $\Omega_+ = \varepsilon_0(3 - 2\eta)/\hbar$. In the vicinity of this frequency there exists a branch of the plasmon spectrum with negative helicity.

In the absence of interband transitions ($\Omega_\pm = 0$) Landau damping regions are bounded by the curve (20) and the axis q.

In formulas (20) perform the limit (13) towards to the nanotube without superlattice. We take into account (8) and

$$\sin x_l = \frac{1}{\Delta}\left[(\mu_0 - \varepsilon_l)(\varepsilon_l + 2\Delta - \mu_0)\right]^{1/2}.$$

Since $qd \ll 1$, from the formulas (20), taking into account the terms of the order q^2 we find

$$\omega_\pm = \Omega_\pm \pm \frac{1}{\hbar} qd\left[(\mu_0 - \varepsilon_l)(\varepsilon_l + 2\Delta - \mu_0)\right]^{1/2} + \frac{1}{2\hbar} q^2 d^2 (\varepsilon_l + \Delta - \mu_0).$$

Passing here to the limit (13), we obtain a parabola

$$\omega_\pm = \Omega_\pm \pm q\upsilon_l \pm \omega_q ,$$

appearing in the formula (14). The maximum speed of the electrons υ_l in the l-th miniband plays the role of the Fermi velocity υ_F of electrons in three-dimensional and two-dimensional electron gas.

The condition of resonant absorption of plasma waves on the tube with a superlattice when $m = 0$ has the form

$$\omega = 2\frac{\Delta}{\hbar}\sin\frac{qd}{2}\sin\left(k_l + \frac{q}{2}\right)d . \qquad (23)$$

In the extreme case $qd \ll 1,\ q \ll k_l,\ k_l d < 1$, it takes the usual form in the theory of waves: $\omega/q = \upsilon_l$ – the phase velocity of the wave, propagating along the tube is equal to the longitudinal velocity of the electrons.

Conclusions

Study of propagation of plasma waves along the tube are very topical problem because it allows to determine the waveguide characteristics of the tube. In addition, it is possible to obtain additional information about the dynamic characteristics of the conduction electrons on the curved surface. The presence of an additional parameter – the curvature of the structure – enriches the picture of wave phenomena, increasing the number of ways to control the properties of the system. In particular, the rotational quantum contains the characteristic of circular motion of electrons on the tube – transverse effective mass. It may differ from the longitudinal mass. Superlattice adds new features to the picture of wave propagation. It is associated with additional parameters – the period and amplitude of the modulating potential. Characteristics of the tube – form and sizes of windows transparency of waves and their spectrum and damping – are sensitive to these parameters. This allows their to determine, by analyzing the properties of waves.

The paper used a simple model spectrum of electrons, simulating the superlattice on the tube. This allowed within the model adopted in the random phase approximation to obtain an exact expression for the polarization operator of the electron gas. As a result, the shape and size of windows transparency for plasma waves were determined in the entire Brillouin zone. They were obtained by analysis of the imaginary part of the polarization operator and with the help of conservation laws. The results can be used in the study of plasma waves in semiconductor superlattices on a base of $\mathrm{Al}_x\mathrm{Ga}_{1-x}\,\mathrm{As}/\mathrm{Ga\,As}$, $\mathrm{InGa\,As}/\mathrm{Ga\,As}$, $\mathrm{In\,As}/\mathrm{Ga\,As}$, $\mathrm{GeSi}/\mathrm{Si}$ and in carbon nanotubes in a metal conduction mode.

1. L.V. Keldysh. Fizika Tverd. Tela, 4, 2265 (1962).
2. L. Esaki, R. Tsu. IBM J. Res. Develop., 14, 61 (1970).
3. J.I. Alferov, Yu.V. Jilyaev, Yu.V. Shmartsev. Fizika Tehn. Poluprovod., 5, 196 (1971).
4. A.Ja. Shick. Fizika Tehn. Poluprovod., 8, 1841 (1974).
5. V.M. Gvozdikov. Fizika Tverd. Tela, 26, 2574 (1984).
6. V.M. Gvozdikov. Fizika Tverd. Tela, 28, 320 (1986).
7. A. Tselis, J. Quinn. Phys. Rev. B, 29, 2021 (1984).
8. A. Tselis, J. Quinn. Phys. Rev. B, 29, 3318 (1984).
9. A.P. Silin. Uspehi Fiz. Nauk, 147, 485 (1985).
10. M. Herman. Semiconductor Superlattices, Akademie-Verlag, Berlin (1986), 240 p.
11. L. Wendler, M.I. Kaganov. JETP letters, 44, 345 (1986).
12. V.N. Lutskiy, M.I. Kaganov, A.Ja. Shick. JETP, 92, 721 (1987).
13. M.I. Kaganov, V.N. Lutskiy, D.V. Posvyanskiy. Fizika Nizk. Temp., 15, 846 (1989).
14. W.-M. Que, G. Kirczenov. Phys. Rev. B, 36, 6596 (1987).
15. F.G. Bass, A.A. Bulgakov, A.P. Tetervov. High-frequency properties of the semiconductor superlattice, Nauka, Moscow (1989), 288 pp.
16. S. Iijima. Nature (London), 354, 56 (1991).
17. L.I. Magarill, A.V. Chaplik, M.V. Entin. Uspehi Fiz. Nauk, 175, 995 (2005).
18. N.A. Poklonskiy, E.F. Kislyakov, G.G. Fedoruk, S.A. Vyrko. Fizika Tverd. Tela, 42, 1911 (2000).
19. J. Lee, H. Kim, S.-J. Kahng, G. Kim, Y.-W. Son, J. Ihm, H. Kato, Z.W. Wang, T. Okazaki, H. Shinohara and Y. Kuk. Nature, 415, 1005 (2002).
20. H.-J. Shin, S. Clair, Y. Kim and M. Kawai. Nat. Nanotech., 4, 567 (2009).
21. Yu. I. Sezonov and Yu.V. Perepelkina. Russ. Journ. Phys. Chemistry B, 3, 247 (2009).
22. A.M. Ermolaev, G.I. Rashba, M.A. Solyanik. Fizika Nizk. Temp., 38, 653 (2012).
23. A.M. Ermolaev and G.I. Rashba. Solid State Commun., 192, 79 (2014).
24. I.O. Kulik. JETP letters, 11, 407 (1970).
25. O.P. Volosnikova, D.V. Zavyalov, S.V. Kruchkov. Proceedings of the XVII International Workshop "Radiational Solid state physics" 645, Sevastopol', 7 (2007).
26. E.A. Pashitskiy, Yu.M. Malozovskiy, A.V. Semenov. Ukr. Fizich. Journ., 36, 889 (1991).
27. A.M. Ermolaev, G.I. Rashba, M.A. Solyanik. Fizika Nizk. Temp., 37, 1033 (2011).