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Transmittance of electromagnetic waves through finite-length layered superconductors in presence of external static magnetic field

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We study the transmission of electromagnetic waves in the presence of external DC magnetic field through a sample of layered superconductor of finite thickness. We show that the external DC magnetic field can be a useful tool to adjust the transmittance of the sample and to vary it over a wide range. Moreover, the right choice of the external DC field provides the total transparency of the sample for the wide range of frequencies.
Keywords: layered superconductors, magnetic field, transmittance.

Изучено прохождение электромагнитных волн в присутствии внешнего постоянного магнитного поля сквозь образец слоистого сверхпроводника конечной толщины. Показано, что внешнее постоянное магнитное поле может быть полезным инструментом для управления коэффициентом пропускания образца и варьирования его в широком диапазоне. Более того, правильный выбор внешнего магнитного поля может обеспечить полную прозрачность образца для широкого диапазона частот.
Ключевые слова: слоистый сверхпроводник, магнитное поле, коэффициент прохождения.

Досліджено проходження електромагнітних хвиль в присутності зовнішнього постійного магнітного поля крізь зразок шаруватого надпровідника скінченної товщини. Показано, що зовнішнє постійне магнітне поле може бути корисним інструментом для управління коефіцієнтом пропускання зразка і варіювання його в широкому діапазоні. Більш того, правильний вибір зовнішнього магнітного поля може забезпечити повну прозорість зразка у широкому діапазоні частот.
Ключові слова: шаруватий надпровідник, магнітне поле, коефіцієнт проходження.

Introduction

High-temperature layered superconductors are interesting materials from different perspectives [1]. The experimental studies for c-axis conductivity in $\mathrm{Bi_2Sr_2CaCu_2O_{8+\delta}}$ (see, e.g., Refs. [2,3]) proved that layered superconductors can be represented as the periodic structures where thin superconducting layers (with thicknesses s of about 0.2 nm) are coupled through thicker dielectric layers (with thicknesses d of about 1.5 nm and a dielectric constant $\varepsilon \sim 16$) via the intrinsic Josephson effect. The anisotropy of the structure causes propagation of the specific electromagnetic excitations, the Josephson plasma waves (JPWs) (see, e.g., Refs. [1,4] and references therein). The frequencies of JPWs are in terahertz range, which makes layered superconductors promising in applications for THz devices (see, e.g., Ref. [5]).

The electrodynamic equations for layered superconductors are nonlinear. The nonlinearity originates from the nonlinear relation $J \propto \sin\varphi$ between the Josephson interlayer current J and the gauge-invariant interlayer phase difference φ of the order parameter. This can lead to a number of non-trivial nonlinear effects accompanying the propagation of JPWs, e.g., slowing down of light [6], self-focusing of terahertz pulses [6,7], excitation of nonlinear waveguide modes [8], as well as self-induced transparency of the slabs of layered superconductors and hysteretic jumps in the dependence of the slab transparency on the wave amplitude [9,10]. The noticeable change in the transparency of the cuprate superconductor when increasing the wave amplitude was recently observed in the experiment Ref. [11], where the excitation of Josephson plasma solitons led to effective decrease of the Josephson resonance frequency.

The external DC magnetic field can be used to control propagation the electromagnetic wave in different media. For example, in the works [12,13] the external DC magnetic field allows to adjust the parameters of the excited surface Josephson plasma waves propagating across the junctions in layered superconductors. In the present work, we concentrate our attention on the transmission of the waves of transverse magnetic (TM) polarization through a sample of layered superconductor in the presence of the external DC magnetic field. We show that the DC magnetic field can be a useful tool to control the transmissivity of the sample of layered superconductor.

Geometry of the problem

We study the transmission of the electromagnetic waves through a sample of layered superconductor of finite thickness D (see Fig. (1)). The coordinate system is chosen in such a way that the crystallographic **ab**-plane of the layered superconductor coincides with the xy-plane, and the **c**-axis is along the z-axis.

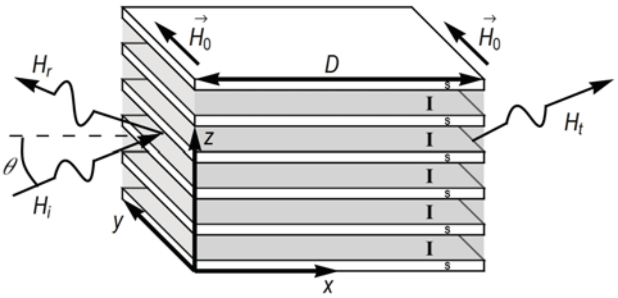

Fig.1. Schematic geometry for the reflection and transmission of waves through the sample of layered superconductor. Here S and I stand for superconducting and insulator layers, respectively, D is the thickness of the sample. The sample is infinite along y and z directions.

The waves are irradiated at an angle θ in xz-plane and have TM-polarization:

$$\vec{E} = \{E_x, 0, E_z\}, \vec{H} = \{0, H_y, 0\}. \quad (1)$$

The external DC magnetic field $\vec{H}_0$ is directed along y-axis in the vacuum regions. We study the relatively small magnetic fields when Josephson vortices do not wholly penetrate inside of the sample.

Fields in the vacuum regions

The electromagnetic field in the vacuum regions to the right and to the left from the sample (see Fig. (1)) is superposition of the DC magnetic field and the field of the incident, reflected and transmitted waves. Using Maxwell equations one can derive following tangential components of the incident and reflected fields in the left region:

$$\begin{aligned} H_y^{left} &= H_i e^{i(k_x x + k_z z - \omega t)} + H_r e^{i(-k_x x + k_z z - \omega t)}, \\ E_z^{left} &= -\frac{k_x}{k}\left(H_i e^{i(k_x x + k_z z - \omega t)} - H_r e^{i(-k_x x + k_z z - \omega t)} \right), \end{aligned} \quad (2)$$

where H_i and H_r are amplitudes of incident and reflected waves, respectively, $k_x = k\cos\theta$, $k_z = k\sin\theta$ are components of the wave vector, $k = \omega / c$ is its module, θ is the incident angle, c is the speed of light.

The tangential components of the transmitted field in the left region are:

$$\begin{aligned} H_y^{right} &= H_t e^{i(k_x(x-D) + k_z z - \omega t)}, \\ E_z^{right} &= -\frac{k_x}{k} H_t e^{i(k_x(x-D) + k_z z - \omega t)}, \end{aligned} \quad (3)$$

where D is the thickness of the sample (see Fig. (1)), H_t is the amplitude of transmitted wave.

The field in the layered superconductor

The electromagnetic field in the layered superconductor is defined by the distribution $\varphi(\vec{r}, t)$ of interlayer gauge-invariant phase difference of the order parameter. This phase difference is governed by a set of coupled sine-Gordon equations [1, 14-18]. In the continual limit and in the main order with respect to the small parameter λ_{ab} / λ_c (for $Bi_2Sr_2CaCu_2O_{8+\delta}$ λ_{ab} / λ_c is about 1/200), the coupled sine-Gordon equations reduce to the following equation:

$$\sin\varphi + \frac{1}{\omega_J^2}\frac{\partial^2 \varphi}{\partial t^2} - \lambda_c^2 \frac{\partial^2 \varphi}{\partial x^2} = 0. \quad (4)$$

Here λ_{ab} and $\lambda_c = c / \omega_J \varepsilon^{1/2}$ are the London penetration depths across and along the layers, respectively, $\omega_J = (8\pi e d J_c / \hbar\varepsilon)^{1/2}$ is the Josephson plasma frequency, J_c is the maximal Josephson current density, and e is the elementary charge, d and ε are the thickness and dielectric conductivity of insulator layers, respectively. We do not take into account the relaxation terms since they are small at low temperatures and do not play an essential role in the phenomena considered here.

The gauge-invariant phase difference of the order parameter $\varphi(\vec{r}, t)$ defines components of the field in the layered superconductor (see e.g., Ref. [1]):

$$\begin{aligned} E_z^s &= \mathcal{H}_0 \frac{1}{\omega_J \sqrt{\varepsilon}} \frac{\partial \varphi}{\partial t}, \\ \frac{\partial H_y^s}{\partial x} &= \frac{\mathcal{H}_0}{\lambda_c}\left[\sin\varphi + \frac{1}{\omega_J^2}\frac{\partial^2 \varphi}{\partial t^2} \right]. \end{aligned} \quad (5)$$

The component E_z of the electric field causes the breakdown of electro-neutrality of the superconducting layers and results in an additional, so-called capacitive, interlayer coupling. In our case $k_z \sim k_x \sim k$, when incident angle is not close to $\pi / 2$, and capacitive coupling can be safely neglected because of the smallness of the parameter $\alpha = R_D^2 \varepsilon / sd$. Here R_D is the Debye length for a charge in a superconductor.

Penetration of external DC magnetic field into the sample

At first we describe how the external DC magnetic field penetrates into the sample of layered superconductor. The DC field exponentially decreases inside the sample. Here we assume that the sample is sufficiently large, $D \gg \lambda_c$, then we neglect interaction between ``tails'' of the field from the right and left boundaries. Using Eq. (4) we can derive expressions for the phase difference in the vicinity of the left and right boundaries:

$$\varphi_0^{left}(\xi) = -4\arctan(e^{-\xi-\xi_0}),$$
$$\varphi_0^{rigth}(\xi) = 4\arctan[e^{\xi-(\delta-\xi_0)}], \qquad (6)$$

where we introduce dimensionless coordinate $\xi = x/\lambda_c$ and normalized thickness of the sample $\delta = D/\lambda_c$. The constant ξ_0 is determined by the magnitude H_0 of external DC field:

$$\xi_0 = \operatorname{arccosh}\frac{1}{h_0}, \quad h_0 = H_0\frac{\pi d\lambda_c}{\Phi_0}, \qquad (7)$$

where $\Phi_0 = \pi c\hbar / e$ is the magnetic flux quantum. The field h_0 is normalized in such a way that Josephson vortices penetrate the superconductor when $h_0 > 1$. So, in this work we study relatively small fields $0 < h_0 < 1$, and the DC field exponentially decreases into the sample at a character distance much less than the thickness of the sample D.

Penetration of electromagnetic field of the wave into the sample

Now we describe the distribution of the fields when the incident electromagnetic wave is penetrating into the sample in the presence of external DC magnetic field. We consider the amplitude of incident wave to be much smaller than the amplitude of the DC magnetic field. In this case the gauge-invariant phase difference can be presented as a sum of three components:

$$\varphi(\xi,z,t) = \varphi_0^{left}(\xi) + \varphi_0^{right}(\xi) + \varphi_v(\xi,z,t), \qquad (8)$$

where first two terms are given by the equations (6). The last term is the small addition, which oscillates with the frequency of the incident wave.

We expand Eq. (4) over a small parameter φ_v and seek it in the form:

$$\varphi_v(\xi,z,t) = a(\xi)e^{i(k_z z-\omega t)}. \qquad (9)$$

Then Eq. (4) reduces to:

$$\left[\frac{2}{\cosh^2(\xi+\xi_0)} + \frac{2}{\cosh^2(\delta+\xi_0-\xi)} + \tilde{\Omega}^2\right]a(\xi) = -\frac{\partial^2 a(\xi)}{\partial \xi^2}, \qquad (10)$$

where Ω is the normalized frequency:

$$\Omega = \frac{\omega}{\omega_J}, \qquad \tilde{\Omega} = \sqrt{\Omega^2 - 1}. \qquad (11)$$

It should be emphasized that the first and second summands in the square brackets in Eq. (10) are independent from each other and essential only in the vicinity of the left and right boundary, respectively. Then the solution of the Eq. (10) can be found analytically:

$$a(\xi) = C_1 e^{i\tilde{\Omega}\xi}[p^2 a_1(\xi) + a_2(\xi) - p\Omega] + C_2 e^{-i\tilde{\Omega}\xi}[a_1(\xi) + p^2 a_2(\xi) + p\Omega], \qquad (12)$$

where

$$a_1(\xi) = \tanh(\xi_0 + \xi) - 1,$$
$$a_2(\xi) = \tanh(\xi_0 + \delta - \xi) - 1, \qquad (13)$$
$$p = \frac{1 + i\tilde{\Omega}}{\Omega}.$$

Using equations (5) and (8), we can find the field in the layered superconductor. The tangential variable components of the field, normalized by the typical field $\Phi_0 / \pi d\lambda_c$, i.e. in the same way as in Eq. (7), are:

$$H_y\frac{\pi d\lambda_c}{\Phi_0} = h_y = \frac{1}{2}a'(\xi)e^{i(k_z z-\omega t)},$$
$$E_z\frac{\pi d\lambda_c}{\Phi_0} = e_z = -\frac{i\Omega}{2\sqrt{\varepsilon}}a(\xi)e^{i(k_z z-\omega t)}. \qquad (14)$$

The transmission coefficient

Matching tangential components of the field in vacuum regions (2) and (3) with the field inside of the sample (14), we can find the unknown constants C_1 and C_2 in Eq. (12). Then we derive the transmission coefficient:

$$T = \frac{|h_t|^2}{|h_i|^2} = \left(1 + \sin^2(\tilde{\Omega}\delta - \phi)\{[\frac{1}{4\Theta} + (\frac{h_0^4\tilde{h}_0^2}{\Omega^4\tilde{\Omega}^2} + 1)\Theta]^2 - 1\}\right)^{-1}, \qquad (15)$$

where

$$\tilde{h}_0 = \sqrt{1-h_0^2}, \quad \Theta = \frac{\Omega\tilde{\Omega}\sqrt{\varepsilon}}{2(\tilde{h}_0^2 + \tilde{\Omega}^2)}\cos\theta,$$
$$\phi = \frac{\pi}{2} - \arctan\left(\frac{1-\tilde{\Omega}^2}{2\tilde{\Omega}} + \frac{\Omega^4\tilde{h}_0}{2\tilde{\Omega}h_0^2}P\right), \qquad (16)$$
$$P = \left[\frac{\Omega^2\varepsilon\cos^2\theta}{\Omega^2 + (\tilde{h}_0^2 - \Omega^2)\varepsilon\cos^2\theta} - \frac{\tilde{h}_0 + \tilde{\Omega}^2}{\tilde{h}_0 + 1}\right]^{-1}.$$

As was mentioned, the DC field h_0 varies from 0 to 1. One can reduce the equation (15) for these two cases. When there is no external field, $h_0 = 0$, we can simplify the equation (15) to the following form:

$$T(h_0 = 0) = \left[1 + \sin^2(\tilde{\Omega}\delta)\left(\frac{1}{4\Theta_0} - \Theta_0\right)^2\right]^{-1}, \qquad \Theta_0 = \frac{\tilde{\Omega}}{\Omega}\frac{\sqrt{\varepsilon}}{2}\cos\theta, \tag{17}$$

When $\cos\theta = \Omega/(\tilde{\Omega}\sqrt{\varepsilon})$, the multiplier of sine is equal to zero and the full transmission occurs regardless the thickness of the sample. Note, that in the absence of DC field the phase ϕ in argument of the sine, Eq. (16), is either zero or π, $\sin\phi_0 = 0$. The deviation of phase ϕ from this value is the main effect from applying DC magnetic field.

If the value of the DC magnetic field is almost critical, $h_0 = 1$, the transmittance takes the form:

$$T(h_0 = 1) = \left[1 + \sin^2(\tilde{\Omega}\delta - \phi_1)\left(\frac{1}{4\Theta_1} - \Theta_1\right)^2\right]^{-1}, \qquad \phi_1 = 2\arctan\tilde{\Omega}, \quad \Theta_1 = \frac{\Omega}{\tilde{\Omega}}\frac{\sqrt{\varepsilon}}{2}\cos\theta, \tag{18}$$

When $\cos\theta = \tilde{\Omega}/(\Omega\sqrt{\varepsilon})$, the transmittance is equal to 1. Thus, we see that for $h_0 = 0$ one can attain full transparency of the sample by choosing the angle for frequencies $\Omega > (1-\varepsilon^{-1})^{-1/2}$, while for $h_0 = 1$ the full transparency can be achieved for any prescribed frequency $\Omega > 1$.

Analysis of results

The dependence of the transmission coefficient on the thickness δ in equation (15) is only via $\sin(\tilde{\Omega}\delta - \phi)$. When the sine multiplier is equal to zero, the sample is transparent. In the absence of external DC magnetic field (see Eq. (17)) the sample is fully transparent, when the thickness is the integer number of the half-wavelengths,

$$\delta = \pi k/\tilde{\Omega}, \quad k = 1, 2... \tag{19}$$

If we turn on the DC field (see Eq. (15) or (18)), the sine acquires a new DC depending term ϕ in the argument. The DC magnetic field shifts the maximum only, while the periodicity of the function $T(\delta)$ remains the same. This shift is shown in the Fig. 2 by the arrow.

The variation of transmittance can be significantly increased by changing the DC field. To demonstrate this, we plot Fig. 3. The upper and lower curves represent the

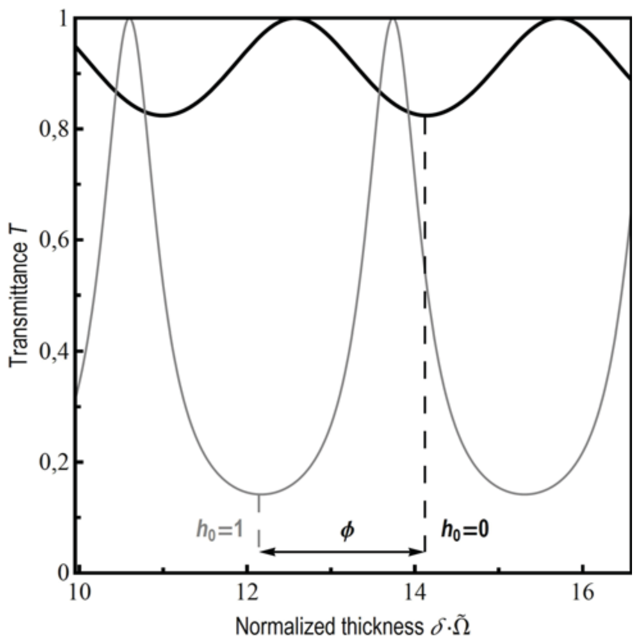

Fig. 2. Transmittance T versus normalized thickness ($\delta = D/\lambda_c$) in the absence of DC magnetic filed (thick black curve, h_0=0) and in the field h_0=1 (thin gray curve). The arrow shows shift of minima with applying of the DC magnetic field. Other parameters: Ω=1.2, $\theta = \pi/4$, $\lambda_c = 4\cdot10^{-3}$ cm, λ_{ab} = 2000 Å, $\omega_J/2\pi$ = 0.3 THz, ε=16.

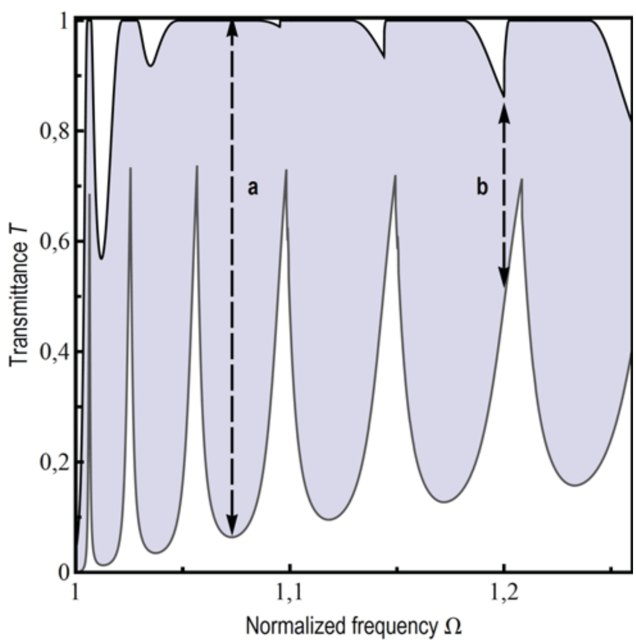

Fig. 3. Transmission T versus normalized frequency Ω. The gray region shows the variation of transmittance when changing the external DC magnetic field, the upper black and bottom gray curves show the maximum and minimum possible transmittance, respectively. Other parameters: $\theta = \pi/4$, δ=30, $\lambda_c = 4\cdot10^{-3}$ cm, λ_{ab} = 2000 Å, $\omega_J/2\pi$ = 0.3 THz, ε=16.

transmittance maximized and minimized over the DC field amplitude as a function of frequency Ω. The gray intermediate region between the upper and lower curves shows the range of transmittance variation.

As one can see from Fig. 3, the range of transmittance variation depends significantly on the choice of the frequency. For example, we can compare the ranges marked by the a and b dashed lines (see Fig. 3). For the frequency corresponding to the line a, one can vary the transmittance nearly from zero to one, while the range is much smaller for the line b and one even cannot attain the full transmission. So the choice of the wave frequency defines the range of transmittance variation. In order to obtain the wide range, one should use relatively small frequencies, but not very close to the Josephson plasma frequency ω_j (see Fig. 3).

Conclusions

In this paper, we have studied theoretically the transmission of transverse magnetic waves through a finite sample of layered superconductor in the presence of external DC magnetic field. The dependence of the transmittance on the thickness of the sample is periodical. The DC field effectively changes the thickness and shifts the maxima of transmittance, but does not affect the periodicity. Also we show that the choice of the wave frequency defines the range of transmittance variation when changing the DC field. Thus, with the right choice of parameters the transmittance can be adjusted by the external DC magnetic field in a wide range.

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